Mathematical Snowflakes Part II

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In my last talk I discussed the following:

1. A general definition for autonomous deterministic dynamical systems.
2. Finite autonomous deterministic (FAD) dynamical systems can be represented by directed graphs.
3. Gave an intuitive description of what these graphs can look like (some look almost like snowflakes).
4. A brief introduction to Koopman operator theory and the Koopman mode decomposition.

I then asked the following questions:

1. How can the Koopman view be used to characterize FAD dynamical systems?
2. Is there any interesting structure of the Koopman operator or its spectrum in the special case of FAD dynamical systems?
3. Can you give a description of some eigenfunctions and how they can be interpreted?

In my next talk I plan to give a brief review of FAD dynamical systems, answer the above questions, and maybe explore a little more of the mathematical relationship between Koopman and graph theoretic concepts in this context.

If you would like to join for part II, you missed part I, and you are not very familiar with Koopman operator theory, I have attached a light introduction. Additionally, if you are not familiar with a general and precise definition of dynamical systems, I have attached a research article in which the first section (titled "introduction") would also be a quick and helpful read.

I'd also like to change the name of these "mathematical snowflakes" to "arboretums"; this is because these mathematical objects consist of collections of graphs called trees (in particular, in-trees, trees (graph theory)), which are grouped into circles via periodic orbits whose nodes

are the roots of the trees. In other words, we have a tree garden - arboretum. I have attached a randomly generated example of a connected arboretum (all pairs of nodes have a path between them, without worrying about direction - this is equivalent to saying that we have only 1 periodic orbit).In this example, the central periodic orbit has a period of 11, there are more than 70 states (nodes), and the largest trees extend no more than three edges past their roots.

Any knowledge on spectral graph theory or graph partitions would be helpful to generate discussion at the end of the meeting but by no means necessary to understand any core content.

Location and Time

Engineering II 2319 ME conference room @ 6pm

Pre-requisites

Linear algebra and freshman calculus

Reference

1. Giunti, Marco, and Claudio Mazzola. "Dynamical systems on monoids: Toward a general theory of deterministic systems and motion." Methods, models, simulations and approaches towards a general theory of change. 2012. 173-185.

2. Arbabi, Hassan. "Introduction to Koopman operator theory of dynamical systems." (2018).